Unit 4: Mathematics for Engineering Technicians

Unit code: A/600/0253

QCF Level 3: BTEC National

Credit value: 10

Guided learning hours: 60

Aim and purpose

This unit aims to give learners a strong foundation in mathematical skills. These skills will help them to successfully complete many of the other units within the qualification.

Unit introduction

One of the main responsibilities of engineers is to solve problems quickly and effectively. This unit will enable learners to solve mathematical, scientific and associated engineering problems at technician level. It will also act as a basis for progression to study other units both within the qualification, such as *Unit 28: Further Mathematics for Technicians*, and at BTEC Higher National level.

This unit enables learners to build on knowledge gained at GCSE or BTEC First Diploma level and use it in a more practical context for their chosen discipline. Learning outcome 1 will develop learners' knowledge and understanding of algebraic methods, from a look at the use of indices in engineering to the use of the algebraic formula for solving quadratic equations. Learning outcome 2 involves the introduction of the radian as another method of angle measurement, the shape of the trigonometric ratios and the use of standard formulae to solve surface areas and volumes of regular solids. Learning outcome 3 requires learners to be able to represent statistical data in a variety of ways and calculate the mean, median and mode. Finally, learning outcome 4 is intended as a basic introduction to the arithmetic of elementary calculus.

Learning outcomes

On completion of this unit a learner should:

1   Be able to use algebraic methods

2   Be able to use trigonometric methods and standard formulae to determine areas and volumes

3   Be able to use statistical methods to display data

4   Be able to use elementary calculus techniques.

Unit content

1 Be able to use algebraic methods

Indices and logarithms: laws of indices ($a^m \times a^n = a^{m+n}$, $\dfrac{a^m}{a^n} = a^{m-n}$, $(a^m)^n = a^{mn}$), laws of logarithms ($\log A + \log B = \log AB$, $\log A^n = n \log A$, $\log A - \log B = \log \dfrac{A}{B}$) eg common logarithms (base 10), natural logarithms (base e), exponential growth and decay

Linear equations and straight line graphs: linear equations eg $y = mx + c$; straight line graph (coordinates on a pair of labelled Cartesian axes, positive or negative gradient, intercept, plot of a straight line); experimental data eg Ohm's law, pair of simultaneous linear equations in two unknowns

Factorisation and quadratics: multiply expressions in brackets by a number, symbol or by another expression in a bracket; by extraction of a common factor eg $ax + ay$, $a(x + 2) + b(x +2)$; by grouping eg $ax - ay + bx - by$; quadratic expressions eg $a^2 + 2ab + b^2$; roots of an equation eg quadratic equations with real roots by factorisation, and by the use of formula

2 Be able to use trigonometric methods and standard formulae to determine areas and volumes

Circular measure: radian; degree measure to radians and vice versa; angular rotations (multiples of π radians); problems involving areas and angles measured in radians; length of arc of a circle ($s = r\theta$); area of a sector ($A = \frac{1}{2} r^2\theta$)

Triangular measurement: functions (sine, cosine and tangent); sine/cosine wave over one complete cycle; graph of $\tan A$ as A varies from 0° and 360° ($\tan A = \sin A/\cos A$); values of the trigonometric ratios for angles between 0° and 360°; periodic properties of the trigonometric functions; the sine and cosine rule; practical problems eg calculation of the phasor sum of two alternating currents, resolution of forces for a vector diagram

Mensuration: standard formulae to solve surface areas and volumes of regular solids eg volume of a cylinder $= \pi r^2 h$, total surface area of a cylinder $= 2\pi rh + 2\pi r^2$, volume of sphere $= \dfrac{4}{3}\pi r^3$, surface area of a sphere $= 4\pi r^2$, volume of a cone $= \dfrac{1}{3}\pi r^2 h$, curved surface area of cone $= \pi r$ x slant height

3 Be able to use statistical methods to display data

Data handling: data represented by statistical diagrams eg bar charts, pie charts, frequency distributions, class boundaries and class width, frequency table; variables (discrete and continuous); histogram (continuous and discrete variants); cumulative frequency curves

Statistical measurement: arithmetic mean; median; mode; discrete and grouped data

4   Be able to use elementary calculus techniques

Differentiation: differential coefficient; gradient of a curve $y = f(x)$; rate of change; Leibniz notation $(\dfrac{dy}{dx})$; differentiation of simple polynomial functions, exponential functions and sinusoidal functions; problems involving evaluation eg gradient at a point

Integration: integration as reverse of differentiating basic rules for simple polynomial functions, exponential functions and sinusoidal functions; indefinite integrals; constant of integration; definite integrals; limits; evaluation of simple polynomial functions; area under a curve eg $y = x(x - 3)$, $y = x^2 + x + 4$

Assessment and grading criteria

In order to pass this unit, the evidence that the learner presents for assessment needs to demonstrate that they can meet all the learning outcomes for the unit. The assessment criteria for a pass grade describe the level of achievement required to pass this unit.

Grading criteria		
To achieve a pass grade the evidence must show that the learner is able to:	**To achieve a merit grade the evidence must show that, in addition to the pass criteria, the learner is able to:**	**To achieve a distinction grade the evidence must show that, in addition to the pass and merit criteria, the learner is able to:**
P1 manipulate and simplify three algebraic expressions using the laws of indices and two using the laws of logarithms	**M1** solve a pair of simultaneous linear equations in two unknowns	**D1** apply graphical methods to the solution of two engineering problems involving exponential growth and decay, analysing the solutions using calculus
P2 solve a linear equation by plotting a straight-line graph using experimental data and use it to deduce the gradient, intercept and equation of the line	**M2** solve one quadratic equation by factorisation and one by the formula method.	**D2** apply the rules for definite integration to two engineering problems that involve summation.
P3 factorise by extraction and grouping of a common factor from expressions with two, three and four terms respectively		
P4 solve circular and triangular measurement problems involving the use of radian, sine, cosine and tangent functions		
P5 sketch each of the three trigonometric functions over a complete cycle		
P6 produce answers to two practical engineering problems involving the sine and cosine rule		
P7 use standard formulae to find surface areas and volumes of regular solids for three different examples respectively		

Grading criteria		
To achieve a pass grade the evidence must show that the learner is able to:	To achieve a merit grade the evidence must show that, in addition to the pass criteria, the learner is able to:	To achieve a distinction grade the evidence must show that, in addition to the pass and merit criteria, the learner is able to:
P8 collect data and produce statistical diagrams, histograms and frequency curves [IE4]		
P9 determine the mean, median and mode for two statistical problems [IE4]		
P10 apply the basic rules of calculus arithmetic to solve three different types of function by differentiation and two different types of function by integration.		

PLTS: This summary references where applicable, in the square brackets, the elements of the personal, learning and thinking skills applicable in the pass criteria. It identifies opportunities for learners to demonstrate effective application of the referenced elements of the skills.

Key	IE – independent enquirers	RL – reflective learners	SM – self-managers
	CT – creative thinkers	TW – team workers	EP – effective participators

Essential guidance for tutors

Delivery

Before starting this unit, learners should be able to demonstrate proficiency in basic mathematical concepts and the use of an electronic scientific calculator to carry out a variety of functions. As a guide to the level required, tutors should consult Unit 3: Mathematics for Engineering Technicians in the Edexcel BTEC Level 2 First Certificate and First Diploma in Engineering.

The learning outcomes are ordered logically and could be delivered sequentially. The use of algebraic methods is required before further skills can be developed and used within the unit. Much of learning outcome 1 can be practised in pure mathematical terms however, tutors could emphasis where these methods would be applied in an engineering context. Obviously much practise in these methods will prove a valuable foundation for the rest of the unit.

Once learners have mastered most of these methods, learning outcome 2 gives opportunities to apply these skills when solving circular and triangular measurement problems. The application of these skills should reflect the context/area of engineering that learners are studying. Formulae do not need to be remembered but correct manipulation of the relevant formulae is very important in solving these problems. Learners should have plenty of practise when drawing graphs for learning outcome 1 and sketching trigonometric functions in learning outcome 2.

During the delivery of this unit there should be opportunities for learners to use statistical data that they have collected from engineering contexts or situations. It is much better to put statistics, required by learning outcome 3, in an engineering context than use generalities such as learners' height, etc.

Again, for learning outcome 4 opportunities to practise differentiation and integration must be given to ensure learners understand these activities within the range of the content and before they are given assessment activities. The range of these calculus techniques are listed within the content.

Note that the use of 'eg' in the content is to give an indication and illustration of the breadth and depth of the area or topic. As such, not all content that follows an 'eg' needs to be taught or assessed.

Outline learning plan

The outline learning plan has been included in this unit as guidance and can be used in conjunction with the programme of suggested assignments.

The outline learning plan demonstrates one way in planning the delivery and assessment of this unit.

Topic and suggested assignments/activities and/assessment
Whole-class teaching:

Whole-class teaching:

- introduction to the unit content, scheme of work and assessment strategy
- discuss the laws of indices giving examples of each and define a logarithm to any base followed by an explanation of how to convert a simple indicial relationship into a logarithmic relationship and vice versa
- define a common logarithm and show how to work out common logarithms with a calculators (using log key) then lead in sketching the graph of a common logarithmic function.

Individual learner activity:

- tutor-led exercises on the solution of problems involving common logarithms.

Whole-class teaching:

- define a natural (Naperian) logarithm and explain how to use a calculator to evaluate a natural logarithm (using $1n$ key)
- lead the class in sketching the natural logarithmic graph and develop the laws of logarithms with reference to the laws of indices
- discuss the relationship between common logarithms and natural logarithms.

Individual learner activity:

- tutor-led exercises on the use of logarithms and their laws to evaluate expressions in science and technology.

Whole-class teaching:

- recall the basic rules of transposition and explain how to solve simple linear equations before showing how a linear equation can be represented by a straight graph
- explain the significance of the gradient (negative and positive) and intercept for the straight line law and then lead the class in the choice of suitable scales and plotting graphs from given data.

Whole-class teaching:

- recap last week's work on straight line graphs and demonstrate the importance of application of straight line law to experimental data.

Individual learner activity:

- tutor-led exercises in plotting straight line graphs and applying straight line law to experimental data.

Topic and suggested assignments/activities and/assessment

Whole-class teaching:

- demonstrate how to solve a pair of simultaneous linear equations in two unknowns using elimination and substitution and then show how equations can be formed as a result of an engineering application (eg Kirchhoff's Laws)

- explain how to solve a pair of simultaneous linear equations in two unknowns using the graphical method

- explain and demonstrate how to factorise expressions containing two, three and four terms by extraction of a common factor and grouping.

Individual learner activities:

- tutor-led exercises in solution of simultaneous equations

- tutor-led exercises in graphical solution of simultaneous equations

- learner activity involving factorisation of different types of expression.

Whole-class teaching:

- explain factorisation of a quadratic expression and develop to find roots of a quadratic equation

- explain and demonstrate the formula method of solving quadratic equations.

Individual learner activities:

- tutor-led exercises in the solution of quadratic equations by factorisation

- tutor-led exercises in the solution of quadratic equations by using the formula.

Preparation for and carrying out **Assignment 1: Algebraic Methods** (P1, P2, P3, M1, M2).

Whole-class teaching:

- define a radian and explain the relationship between radian and degree, then show how to convert radians to degrees and vice versa

- demonstrate angular rotations and show how to solve problems involving areas and angles measured in radians

- revise trigonometrical ratios (sine, cosine and tangent) and explain the use of a calculator to find different values in degrees and radians

- explain the use of a calculator to construct a table of values from $0°$ to $360°$ (2π) for each of the three functions. Plot graphs of the three functions and demonstrate the use of graphs in evaluation of values of trigonometric ratios for angles between $0°$ to $360°$.

Individual learner activities:

- tutor-led solutions of problems involving radians

- tutor-led solution of problems involving functions (sine, cosine and tangent)

- tutor-led solution of evaluation of values of trigonometric ratios between $0°$ to $360°$.

Topic and suggested assignments/activities and/assessment

Whole-class teaching:

- review previous weeks' work and summarise values of sine, cosine and tangent values (4 quadrant diagrams) then compare features of the three functions (periodic properties)
- explain the use of the sine rule and conditions for solving non right angled triangles.

Individual learner activities:

- tutor-led solution of problems on triangular measurement.
- tutor-led solution of practical problems (electrical and mechanical) involving the sine rule.

Whole-class teaching:

- explain the use of the cosine rule and conditions for use (eg where sine rule cannot be used)
- explain and demonstrate the use of standard formulae to solve problems involving surface areas and volumes of regular solids.

Individual learner activities:

- tutor-led solution of practical problems (electrical and mechanical) involving just the cosine rule and then the use of both the sine and cosine rule together
- tutor-led solution of problems on mensuration.

Preparation for and carrying out **Assignment 2: Trigonometric Methods and Standard Formulae** (P4, P5, P6, P7).

Whole-class teaching:

- explain and demonstrate how statistical information can be displayed
- explain and demonstrate evaluation of mean, median and mode for discrete data
- explain and demonstrate evaluation of mean, median and mode for grouped data.

Individual learner activities:

- tutor-led solution of problems on data collection
- tutor-led evaluation of problems involving mean, median and mode for discrete data
- tutor-led evaluation of problems involving mean, median and mode for grouped data.

Preparation for and carrying out **Assignment 3: Statistical Methods** (P8, P9).

Whole-class teaching:

- explain and introduce differentiation as a measure of the gradient by evaluating various gradients on straight lines and curves
- introduce the idea of rate of change and explain the notation followed by an introduction of the general rule for differentiation and demonstration of use on simple algebraic functions
- review the general rule for differentiation of simple polynomial functions and introduce and demonstrate the rules for exponential and sinusoidal functions.

Individual learner activities:

- tutor-led differentiation of simple algebraic functions
- tutor-led differentiation of exponential and sinusoidal functions.

Topic and suggested assignments/activities and/assessment
Whole-class teaching:
• consolidate all differential coefficients considered so far and explain and demonstrate valuation to find gradients and rates of charge
• explain and introduce the basic rules for integration, the idea of indefinite integration and the constant of integration.
Individual learner activities:
• tutor-led evaluation of problems involving all functions (polynomial, exponential and sinusoidal) using graphical and checking by differentiating
• tutor-led solution of problems on integration of simple polynomial, exponential and sinusoidal functions.
Whole-class teaching:
• introduce definite integration as indefinite integration with the addition of limits
• demonstrate the evaluation of simple polynomial functions and show how integration can be used to evaluate the area under a curve.
Individual learner activities:
• revision documentation on differentiation and integration
• tutor-led evaluation of problems on definite integration.
Preparation for and carrying out **Assignment 4: Calculus Techniques** (P10, D1, D2).
Feedback on all assessment tasks, guidance on remedial action if necessary.
Unit evaluation and close.

Assessment

The assessment strategy used will need to cover all the learning outcomes and associated pass criteria but not necessarily all the topics included in the unit content.

Criterion P1 may be best assessed in the form of a short written test and could possibly also include criterion P3.

P2 could be assessed through an assignment using data from either *Unit 5: Mechanical Principles and Applications* and/or *Unit 6: Electrical and Electronic Principles*, which ideally would be delivered concurrently with this unit. If this not possible, learners should be given a range of data sufficient for them to plot the graph and work out the gradient, intercept and the equation. Data forcing them to draw the line of best fit, as opposed to a set of points directly on the graphical line, might be most appropriate.

For P4, learners could be given a range of different values and assessed by an assignment or a short formal test. The problems given should collectively cover radian, sine, cosine and tangent functions. When considering the content part of this learning outcome it is important that these problems give the learner the opportunity to convert multiples of π radians to degrees and vice versa. The circular measurement problems also need to cover the length of an arc and area of a sector as well as areas and angles measured in radians. Obviously the triangular measurement problems are more basic and only expect application of the three functions.

P5 requires learners to sketch each of the three trigonometric ratios and this is probably best done as a classroom exercise. Similarly, P6 could take the form of a written assignment where learners must produce answers to two practical engineering problems involving the sine and cosine rule (for example calculate the phasor sum of two alternating currents and evaluate the resultant and the angle between two forces).

Criterion P7 requires learners to calculate the surface areas and volumes for three different regular solids. This could be achieved through an assignment or perhaps by combining it with other criteria in a short formal test.

An assignment could be used for P8 where learners collect meaningful data (for example classification of workers within their company) and display this information using different graphical methods (for example bar charts). They also need to produce a histogram and plot frequency curves (for example resistance values of 100 resistors or external diameter of pins).

For P9, learners must provide evidence that they are able to determine and then explain the relevance of the mean, median and mode for a set of discrete and grouped data (for example time taken to produce components on a machine rounded to the nearest ten seconds and the 100 resistor values or diameters of pins from P8). This could be done by an assignment. P10 may be assessed through a short formal test, with learners being given a list of the standard differential coefficients and integrals to use.

For M1, learners will need to provide evidence that they can solve a pair of simultaneous linear equations in two unknowns (for example equations formed after the application of Kirchhoff's laws, power transmitted for different belt tensions in a mechanical system). It would be appropriate to use the same assessment method and instrument as P2, possibly combining these two criteria as one assessment activity.

M2 could also be assessed by assignment as it requires learners to evaluate the roots of a quadratic equation by factorisation and by the formula method (for example evaluation of an equation formed after the realisation of a practical situation).

Both the distinction criteria could be assessed through a written assignment. For D1, learners need to apply graphical methods to the solution of two engineering problems involving exponential growth and decay (for example growth of voltage in a capacitor, radioactive decay, application of Taylor's tool life equation $C = VT^n$) and then analyse the results by applying the appropriate method of differential calculus to check the results.

D2 requires learners to demonstrate that they can accurately evaluate two engineering problems involving definite integration (for example area under a velocity-time graph, area under a voltage-current graph).

Programme of suggested assignments

The table below shows a programme of suggested assignments that cover the pass, merit and distinction criteria in the assessment and grading grid. This is for guidance and it is recommended that centres either write their own assignments or adapt any Edexcel assignments to meet local needs and resources.

Criteria covered	Assignment title	Scenario	Assessment method
P1, P2, P3, M1, M2	Algebraic Methods	A written activity requiring learners to complete five tasks, one for each of the criteria.	A report containing written solutions to each of the five tasks carried out under controlled conditions.
P4, P5, P6, P7	Trigonometric Methods and Standard Formulae	A written activity requiring learners to use trigonometric methods and standard formula to determine areas and volumes.	A report containing the results of calculations, and graphic evidence to support the use of trigonometric methods and standard formula for the determination of areas and volumes.
P8, P9	Statistical Methods	A written activity requiring learners to collect and display data using different graphical methods, also evaluate the mean, median and mode for a set of discrete and grouped data.	A report containing bar charts, pie charts and the results of calculations to determine the mean, median and mode for a set of discrete and grouped data
P10, D1, D2	Calculus Techniques	A written activity requiring learners to produce calculations, graphical solutions and analysis to demonstrate use of calculus techniques.	A report containing the solutions to calculations, graphs and analysis of several calculus techniques. Carried out under controlled conditions.

Links to National Occupational Standards, other BTEC units, other BTEC qualifications and other relevant units and qualifications

This unit forms part of the BTEC Engineering sector suite. This unit has particular links with the following unit titles in the Engineering suite:

Level 1	Level 2	Level 3
	Mathematics for Engineering Technicians	Electrical and Electronic Principles
		Mechanical Principles and Applications
		Advanced Mechanical Principles and Application
		Further Mathematics for Technicians

Essential resources

Learners will need to possess an electronic scientific calculator and have access to software packages that support understanding of the principles and their application to engineering.

Employer engagement and vocational contexts

There is a range of organisations that may be able to help centres engage and involve local employers in the delivery of this unit, for example:

- Work Experience/Workplace learning frameworks – Centre for Education and Industry (CEI, University of Warwick) – www.warwick.ac.uk/wie/cei/

- Learning and Skills Network – www.vocationallearning.org.uk

- Network for Science, Technology, Engineering and Maths Network Ambassadors Scheme – www.stemnet.org.uk

- National Education and Business Partnership Network – www.nebpn.org

- Local, regional Business links – www.businesslink.gov.uk

- Work-based learning guidance – www.aimhighersw.ac.uk/wbl.htm

Indicative reading for learners

Textbooks

Boyce A, Cooke E, Jones R and Weatherill B – *BTEC Level 3 National Engineering Student Book* (Pearson, 2010) ISBN 9781846907241

Boyce A, Cooke E, Jones R and Weatherill B – *BTEC Level 3 National Engineering Teaching Resource Pack* (Pearson, 2010) ISBN 9781846907265

Bird J – *Engineering Mathematics* (Elsevier Science & Technology, 2007) ISBN 9780750685559

Fuller A, Greer A, Taylor G W – *BTEC National Mathematics for Technicians* (Nelson Thornes, 2004) ISBN 9780748779499

Tooley M and Dingle L – *BTEC National Engineering, 2nd Edition* (Elsevier Science & Technology, 2007) ISBN 9780750685214

Delivery of personal, learning and thinking skills

The table below identifies the opportunities for personal, learning and thinking skills (PLTS) that have been included within the pass assessment criteria of this unit.

Skill	When learners are ...
Independent enquirers	analysing and evaluating statistical information, judging its relevance and value.

Although PLTS are identified within this unit as an inherent part of the assessment criteria, there are further opportunities to develop a range of PLTS through various approaches to teaching and learning.

Skill	When learners are ...
Creative thinkers	trying out alternatives or new solutions to mathematics problems
Reflective learners	reviewing progress when solving problems during the learner's activities and acting on the outcomes to make corrections to understanding/solutions
Team workers	collaborating with others when working on investigative group work to achieve a valid solution
Self-managers	organising time and resources, prioritising actions.

● Functional Skills – Level 2

Skill	When learners are …
Mathematics	
Understand routine and non-routine problems in a wide range of familiar and unfamiliar contexts and situations	solving routine electrical and mechanical problems set within engineering contexts and situations
Identify the situation or problem and the mathematical methods needed to tackle it	recognising the relevant parameters and formulae to be applied to given electrical and mechanical situations
Select and apply a range of skills to find solutions	selecting and applying formulae to solve electrical/mechanical problems in engineering
Use appropriate checking procedures and evaluate their effectiveness at each stage	checking the results of solutions to electrical and mechanical problems to evaluate their effectiveness and reality at each stage of the calculation
English	
Speaking and listening – make a range of contributions to discussions and make effective presentations in a wide range of contexts	speaking with and listening to peers and supervisors to establish an understanding of mathematical concepts and issues in engineering
Reading – compare, select, read and understand texts and use them to gather information, ideas, arguments and opinions	selecting, reading and using appropriate mathematical data sources to solve engineering problems
Writing – write documents, including extended writing pieces, communicating information, ideas and opinions, effectively and persuasively	taking notes and solving engineering mathematical problems to communicate accurate solutions effectively.