mathtutor
algebra

Simplifying fractions

The ability to simplify fractions and to write them in equivalent forms is an essential mathematical skill required of all engineers and physical scientists. This unit explains how these processes are carried out.

In order to master the techniques explained here it is vital that you undertake plenty of practice exercises so that they become second nature. To help you to achieve this, the unit includes a number of such exercises.

After reading this text, and/or viewing the video tutorial on this topic, you should be able to:

- simplify algebraic expressions by cancelling common factors

Contents

1. Introduction

When we come across fractions one of the things we often have to do is to study them to see if we can put them in a simpler form. In fact, we look to see if we can put them in a form that might be called their **lowest terms** or **simplest form**.

For instance, suppose we have a fraction $\frac{12}{36}$. This is not in its lowest terms. 12 will divide into both the numerator, 12, and the denominator 36. So, we can divide both numerator and denominator by 12 to give

$$\frac{\not{12}^{\,1}}{\not{36}_{\,3}} = \frac{1}{3}$$

The two fractions $\frac{12}{36}$ and $\frac{1}{3}$ have the same value; they are **equivalent fractions**.

We want to carry out similar operations with algebraic expressions. Instead of looking for numbers which will divide into both the numerator and the denominator, we now look for algebraic expressions which will divide into both.

2. Some introductory examples

Example

Suppose we wish to write

$$\frac{3x^3}{x^5}$$

in its simplest form.

Clearly there are x terms in both the numerator and the denominator. Because x^5 can be considered as $x^3 \times x^2$ we observe a common factor of x^3 in both numerator and denominator.

$$\frac{3x^3}{x^5} = \frac{3x^3}{x^3x^2}$$

Now we can divide top and bottom by x^3 to remove this common factor.

$$\frac{3x^3}{x^5} = \frac{3}{x^2}$$

With practice you will be able to miss out the middle step and go straight from $\frac{3x^3}{x^5}$ to $\frac{3}{x^2}$

Example

Suppose we wish to write

$$\frac{x^3y^3}{x^2y}$$

in its simplest form.

We look for expressions which are common to both the top and the bottom. Because x^3 can be considered as $x \times x^2$ we observe that there is a common factor of x^2. Similarly because y^3 can be considered as $y \times y^2$ we observe that there is also a common factor of y.

We can divide top and bottom by x^2, and then by y to give

$$\frac{x^3y^3}{x^2y} = \frac{xy^2}{1} = xy^2$$

Example

Suppose we wish to write

$$\frac{-16x^2y^2}{4x^3y^2}$$

in its simplest form.

Because $-16 = -4 \times 4$ we see there is a common factor of 4 in both numerator and denominator. Note also that both x^2 and y^2 are common factors. We can divide top and bottom, in turn, by each of these common factors:

$$\frac{-16x^2y^2}{4x^3y^2} = \frac{-4}{x}$$

Exercise 1
Express each of the following fractions in its simplest form.

a) $\dfrac{12x^2}{x^5}$ b) $\dfrac{5x^3y^2}{10xy^4}$ c) $\dfrac{4x^8}{2x^6}$ d) $\dfrac{9y^2}{3y}$

e) $\dfrac{4x^2y^{10}}{12x^4y^5}$ f) $\dfrac{3(x+y)^5}{12(x+y)}$ g) $\dfrac{-9xy}{27x^2}$ h) $\dfrac{-14xy^5}{2x^3y^2}$

3. Examples requiring factorisation of the numerator or denominator

Example

Suppose we wish to write

$$\frac{x^2-2xy}{x}$$

in its simplest form. In this Example the common factor may not be obvious. We start by asking is there a common factor in the numerator. In both terms x^2 and $2xy$ there is a factor of x, so we can factorise the numerator.

$$\frac{x^2-2xy}{x} = \frac{x(x-2y)}{x}$$

In this form we can see that there is a common factor of x in both the numerator and the denominator. We can divide top and bottom by x to remove this:

$$\frac{x^2-2xy}{x} = \frac{x(x-2y)}{x} = \frac{x-2y}{1} = x-2y$$

So $\dfrac{x^2-2xy}{x}$ is equivalent to $x-2y$.

Example

Suppose we wish to write

$$\frac{x^6-7x^5+4x^8}{x^2}$$

in its simplest form.

As in the previous example we look for common factors in the numerator. Observe there is a common factor of x^5 and so we factorise the numerator as follows:

$$\frac{x^6 - 7x^5 + 4x^8}{x^2} = \frac{x^5(x - 7 + 4x^3)}{x^2}$$

There is clearly a common factor of x^2 in the numerator and the denominator, and so dividing top and bottom by this factor we find

$$\frac{x^6 - 7x^5 + 4x^8}{x^2} = \frac{x^5(x - 7 + 4x^3)}{x^2} = \frac{x^3(x - 7 + 4x^3)}{1} = x^3(x - 7 + 4x^3)$$

There is no need to remove the brackets in the answer.

Example

Suppose we wish to write

$$\frac{x+1}{x^2 + 3x + 2}$$

in its simplest form.

There are no obvious common factors. But, the denominator is a quadratic and so we try to factorise it. It may turn out that one of the factors is the same as the term at the top. So factorising the denominator

$$\frac{x+1}{x^2 + 3x + 2} = \frac{(x+1)}{(x+2)(x+1)}$$

In this form we see a common factor of $(x+1)$. Dividing top and bottom by this factor removes it:

$$\frac{x+1}{x^2 + 3x + 2} = \frac{\cancel{(x+1)}^{\,1}}{(x+2)\,\cancel{(x+1)}_{\,1}} = \frac{1}{x+2}$$

Example

Suppose we wish to write

$$\frac{a^2 - 11a + 30}{a - 5}$$

in its simplest form. Again, there is a quadratic which we can try to factorise:

$$\frac{a^2 - 11a + 30}{a - 5} = \frac{(a-6)(a-5)}{(a-5)}$$

Notice the common factor of $(a-5)$. We can divide top and bottom by this factor to remove it.

$$\begin{aligned} \frac{a^2 - 11a + 30}{a - 5} &= \frac{(a-6)\,\cancel{(a-5)}^{\,1}}{\cancel{(a-5)}_{\,1}} \\ &= a - 6 \end{aligned}$$

4. A mistake to be avoided

Dividing top and bottom by common factors is often loosely referred to as *cancelling* common factors, because, as we have seen, we cancel them out during our working. A common mistake that students sometimes make is to cancel out terms which ought not to be cancelled. We must not be tempted to cancel numbers straightaway. We can only cancel common **factors**. For example, we have seen students take an expression such as

$$\frac{3x^2+10x+3}{x+3}$$

and simply cancel out all the 3's. This would certainly be incorrect. Let us look at a numerical example to see why.

Suppose we have $\dfrac{5+3}{3+1}$. Cancelling the 3's we would find

$$\frac{5+\cancel{3}^{\,1}}{{}_1\cancel{3}+1}=\frac{6}{2}=3$$

However, the true value is obtained correctly as

$$\frac{5+3}{3+1}=\frac{8}{4}=\frac{\cancel{8}^{\,2}}{\cancel{4}_{1}}=2$$

We see that simply cancelling 3's leads to an incorrect result. Never do this. You must only cancel common factors.

The correct way to handle $\dfrac{3x^2+10x+3}{x+3}$ is to factorise the numerator and then cancel any common factors:

$$\frac{3x^2+10x+3}{x+3}=\frac{(3x+1)(x+3)}{(x+3)}=\frac{(3x+1)\,\cancel{(x+3)}^{1}}{\cancel{(x+3)}_{1}}=3x+1$$

Example

Suppose we wish to write

$$\frac{6x^3-7x^2-5x}{2x+1}$$

in its simplest form. Observe the common factor of x in each of the terms in the numerator. We proceed as follows.

$$\frac{6x^3-7x^2-5x}{2x+1}=\frac{x(6x^2-7x-5)}{2x+1}=\frac{x(3x-5)\,\cancel{(2x+1)}^{1}}{\cancel{(2x+1)}_{1}}=x(3x-5)$$

Again, it is better to leave this answer in its factorised form.

Example

Suppose we wish to write

$$\frac{x^3-1}{x-1}$$

in its simplest form. It is not obvious how to proceed here.

However the numerator will factorise to

$$x^3 - 1 = (x - 1)(x^2 + x + 1)$$

You should remove the brackets yourself to check this. Then,

$$\frac{x^3 - 1}{x - 1} = \frac{{}^1\cancel{(x - 1)}\,(x^2 + x + 1)}{\cancel{(x - 1)}_1} = x^2 + x + 1$$

Exercise 2

Express each of the following fractions in its simplest form.

a) $\dfrac{2x + 4}{12}$ b) $\dfrac{xy - 3x}{x^3}$ c) $\dfrac{x^2y + y^2x}{x^2y^2}$ d) $\dfrac{x^3 - 2x^2 + 5x}{x^4}$

e) $\dfrac{5x^2 + 25x}{100x^3}$ f) $\dfrac{x + 2}{x^2 - x - 6}$ g) $\dfrac{x^2 + 5x}{x + 5}$ h) $\dfrac{x + 2y}{x^2 + 3xy + 2y^2}$

Answers

Exercise 1

a) $\dfrac{12}{x^3}$ b) $\dfrac{x^2}{2y^2}$ c) $2x^2$ d) $3y$

e) $\dfrac{y^5}{3x^2}$ f) $\dfrac{(x + y)^4}{4}$ g) $\dfrac{-y}{3x}$ h) $\dfrac{-7y^3}{x^2}$

Exercise 2

a) $\dfrac{x + 2}{6}$ b) $\dfrac{y - 3}{x^2}$ c) $\dfrac{x + y}{xy}$ d) $\dfrac{x^2 - 2x + 5}{x^3}$

e) $\dfrac{x + 5}{20x^2}$ f) $\dfrac{1}{x - 3}$ g) x h) $\dfrac{1}{x + y}$